THEORY OF CREEP OF METALLIC MATERIALS ALLOWING FOR THE EFFECT OF THE SIMILARITY PHASE OF THE DEVIATORS

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Namestnikov [1] was the first to experimentally determine the existence of a departure from the similarity laws of stress and strain deviators under creep conditions. This problem was explored further in [2-7], while in [8] we discussed the significance of the similarity phase of the deviators in the theory of plasticity. Below, the arguments made in [8] are extended to creep theory.

1. Novozhilov [9] proposed a formula to connect the two coaxial deviators a_{ij}' and b_{ij}':

$$
\begin{gathered}
a_{ij}' = 2G\left[\frac{\cos(3\beta+\omega)}{\cos 3\beta} b_{ij}' - \frac{\sqrt{6}}{\sqrt{b_2}}\frac{\sin\omega}{\cos 3\beta}\left(b_{ik}'b_{kj}' - \frac{1}{3}b_2\delta_{ij}\right)\right],\\
2G = f_1(\sqrt{b_2}, \beta),\ \omega = f_2(\sqrt{b_2}, \beta),\ \omega = \alpha - \beta,\\
\beta = -\frac{1}{3}\arcsin\frac{\sqrt{6}\, b_3}{b_2^{3/2}},\ -\frac{\pi}{6} \leqslant \beta \leqslant \frac{\pi}{6},\\
\alpha = -\frac{1}{3}\arcsin\frac{\sqrt{6}\, a_3}{a_2^{3/2}},\ -\frac{\pi}{6} \leqslant \alpha \leqslant \frac{\pi}{6},\\
a_2 = a_{ij}'a_{ij}',\ a_3 = a_{ik}'a_{kj}'a_{ji}',\\
b_2 = b_{ij}'b_{ij}',\ b_3 = b_{ik}'b_{kj}'b_{ji}'.
\end{gathered}
\tag{1.1}
$$

Here, ω is the similarity phase of the deviators a_{ij}' and b_{ij}', characterizing the extent to which the deviators depart from the similarity law. It was recommended in [10] that (1.1) be modified as follows

$$
\begin{gathered}
a_{ij}' = \sqrt{a_2}\left[\sin\alpha\frac{\partial(\sqrt{b_2}\sin\beta)}{\partial b_{ij}'} + \cos\alpha\frac{\partial(\sqrt{b_2}\cos\beta)}{\partial b_{ij}'}\right],\\
\sqrt{a_2} = \Phi_1(\sqrt{b_2}, \beta),\ \omega = \Phi_2(\sqrt{b_2}, \beta).
\end{gathered}
$$

A somewhat different form was proposed in [11] to represent two coaxial deviators:

$$
\begin{gathered}
a_{ij}' = W\left[\frac{1}{\sqrt{b_2}}\frac{\partial\sqrt{b_2}}{\partial b_{ij}'} + \operatorname{tg}\omega\frac{\partial\beta}{\partial b_{ij}'}\right],\\
W = \sqrt{a_2}\sqrt{b_2}\cos\omega,\ W = \Phi_3(\sqrt{b_2}, \beta),\ \omega = \Phi_4(\sqrt{b_2}, \beta).
\end{gathered}
$$

The below representation was recommended in [12]

$$
a_{ij}' = W\left[(1 - \operatorname{tg}3\beta\operatorname{tg}\omega)\frac{b_{ij}'}{b_2} - \frac{\operatorname{tg}3\beta\operatorname{tg}\omega}{b_3}\left(b_{ik}'b_{kj}' - \frac{1}{3}b_2\delta_{ij}\right)\right].
$$

Finally, Kadashevich et al. [8] proposed the relation

$$
a_{ij}' = \sqrt{a_2}\left[\cos\omega\frac{b_{ij}'}{\sqrt{b_2}} - \sin\omega\frac{b_{ij}^*}{\sqrt{b_2}}\right],\ \sqrt{a_2} = \Phi_1(\sqrt{b_2}, \beta),\ \omega = \Phi_2(\sqrt{b_2}, \beta),
\tag{1.2}
$$

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where

$$b_{ij}^{*}=\frac{\sqrt{6}}{\cos 3\beta}\left[\frac{b_{ik}'b_{kj}'}{\sqrt{b_2}}-\frac{\sqrt{b_2}\delta_{ij}}{3}+\frac{\sin 3\beta}{\sqrt{6}}b_{ij}'\right].$$

Formula (1.2) can be rewritten in the more compact form

$$a_{ij}'=\sqrt{a_2}\,[\cos\omega\eta_{ij}-\sin\omega\mu_{ij}].$$

Here, $\eta_{ij} = \partial\sqrt{b_2}/\partial b_{ij}'$ and $\mu_{ij} = -\sqrt{b_2}\,\partial\beta/\partial b_{ij}'$ are the normalized gradients of the invariants $\sqrt{b_2}$ and β, having the obvious properties: $\eta_{ij}\eta_{ij} = 1$, $\mu_{ij}\mu_{ij} = 1$, $\eta_{ij}\mu_{ij} = 0$. If we require that

$$\operatorname{tg}\omega=\frac{\partial\Phi_1(\sqrt{b_2},\beta)/\partial\beta}{\sqrt{b_2}\,\partial\Phi_1(\sqrt{b_2},\beta)/\partial\sqrt{b_2}},$$

then Eqs. (1.2) will be of the gradient type. If we assume that $\Phi_1(\sqrt{b_2}, \beta) = \Phi_1(\sqrt{b_2}\varphi(\beta))$, then $\tan\omega = \varphi'(\beta)/\varphi(\beta)$. It should be noted that in [11, 12]

$$a_{ij}'=d\varepsilon_{ij}^{i}/dt,\ b_{ij}'=\sigma_{ij}',$$

while in [8]

$$a_{ij}'=d\varepsilon_{ij}^{i},\quad b_{ij}'=\sigma_{ij}'-\rho_{ij}.$$

Here ε_{ij}^{i} is the deviator of the inelastic strains; σ_{ij}' is the deviator of the stress tensor; ρ_{ij} is the deviator of the tensor of the microstresses.

We can also use (1.2) to find more general relations that better correspond to the empirical creep data. In particular,

$$a_{ij}'=d\varepsilon_{ij}^{i}/d\mu,\ b_{ij}'=\sigma_{ij}'-\rho_{ij},\ \rho_{ij}=k(\lambda,\beta)\,\varepsilon_{ij}^{i},$$
$$d\mu/d\lambda=1/f(\lambda,\lambda^{\cdot},\mu),\ d\lambda=\left(d\varepsilon_{ij}^{i}d\varepsilon_{ij}^{i}\right)^{1/2},\ \lambda^{\cdot}=d\lambda/dt.$$

It is easily seen that, given such a representation, it is possible to obtain different variants of the theory of plasticity and creep.

2. We take the following as the simplest theory variant accounting for the similarity phase of the deviators

$$b_{ij}'=\sigma_{ij}'-\rho_{ij}\equiv\tau_{ij},\ \Phi_1(\sqrt{b_2},\beta)=\Phi_1(\sqrt{b_2}\,\varphi(\beta)),\ f(\lambda,\lambda^{\cdot},\mu)=Q(\lambda,\lambda^{\cdot})$$

(τ_{ij} is the deviator of the tensor of the active stresses). Then we can write (1.2) in the form

$$d\varepsilon_{ij}^{i}/d\mu=\Phi_1(\sqrt{\tau_2}\,\varphi(\beta))\,L_{ij},\ d\mu/d\lambda=1/Q(\lambda,\lambda^{\cdot}),$$

where

$$L_{ij}=\cos\omega\cdot\tau_{ij}/\sqrt{\tau_2}-\sin\omega\cdot\tau_{ij}^{*}/\sqrt{\tau_2},\ \tau_2=\tau_{ij}\tau_{ij},$$

or

$$Q(\lambda,\lambda^{\cdot})\,d\varepsilon_{ij}^{i}/d\lambda=\Phi_1(\sqrt{\tau_2}\,\varphi(\beta))\,L_{ij}.$$

It follows from this that $Q(\lambda, \lambda^{\cdot}) = \Phi_1(\sqrt{\tau_2}\varphi(\beta))$ or $\lambda^{\cdot} = F(\sqrt{\tau_2}\varphi(\beta), \lambda)$. Considering that $d\varepsilon_{ij}^{i}/d\lambda = (d\varepsilon_{ij}^{i}/dt)(1/\lambda^{\cdot})$, we obtain $d\varepsilon_{ij}^{i}/dt = \lambda^{\cdot}L_{ij}$ or, finally, $d\varepsilon_{ij}^{i}/dt = F(\sqrt{\tau_2}\varphi(\beta), \lambda)L_{ij}$.

In [11], the function $\varphi(\beta) = 1 + k_1 \sin 3\beta + k_2 \sin^2 3\beta$. In [13], an approximation was proposed with the use of the exponential function $\varphi(\beta) = \exp(k_1 \sin 3\beta + k_2 \sin^2 3\beta)$, while the

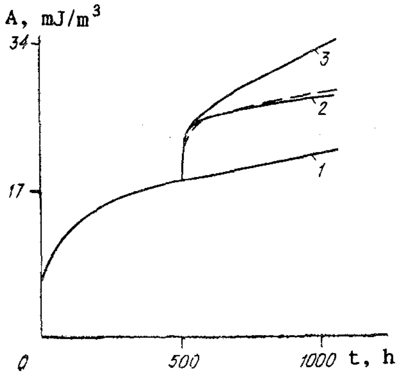

Fig. 1

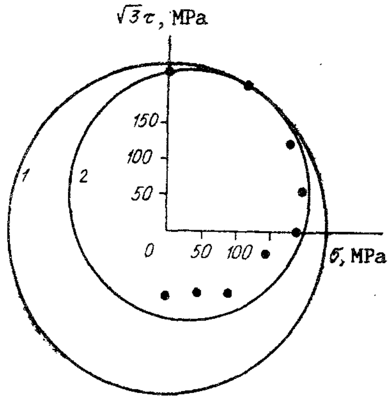

Fig. 2

following relation was proposed in [8]

$$\varphi(\beta)=\begin{cases}\exp\,(k_1(\cos 6\beta-1)),\ \beta\leqslant 0,\\ \exp\,(k_2(\cos 6\beta-1)),\ \beta\geqslant 0.\end{cases} \tag{2.1}$$

3. To demonstrate the possibilities of the theory, accounting for the coaxiality of the deviators, we will consider examples involving the deformation of materials uncer creep conditions. We set $\tau_{ij} = \sigma_{ij}' - k\varepsilon_{ij}^{i}$, $F(\sqrt{\tau_2}\varphi(\beta), \lambda) = B[\sqrt{\tau_2}\varphi(\beta) + k\lambda]^n\lambda^m$. Then we write the working formulas as

$$\begin{gathered}
d\varepsilon_{ij}^{i}/dt = B\left[\sqrt{\tau_2}\,\varphi(\beta)+k\lambda\right]^n\lambda^m L_{ij},\\
\tau_{ij}=\sigma_{ij}'-k\varepsilon_{ij}^{i},\ \sigma_{ij}'=\sigma_{ij}-\frac{1}{3}\,\sigma_{ii}\delta_{ij},\\
L_{ij}=\cos\omega\,\frac{\tau_{ij}}{\sqrt{\tau_2}}-\sin\omega\,\frac{\tau_{ij}^{*}}{\sqrt{\tau_2}},\ \operatorname{tg}\omega=\varphi'(\beta)/\varphi(\beta),
\end{gathered} \tag{3.1}$$

$$\begin{gathered}
\tau_{ij}^{*}=\frac{\sqrt{6}}{\cos 3\beta}\left[\frac{\tau_{ik}\tau_{kj}}{\sqrt{\tau_2}}-\frac{\sqrt{\tau_2}\,\delta_{ij}}{3}+\frac{\sin 3\beta}{\sqrt{6}}\,\tau_{ij}\right],\\
\tau_2=\tau_{ij}\tau_{ij},\ \tau_3=\tau_{ik}\tau_{kj}\tau_{ji},\ d\lambda=\left(d\varepsilon_{ij}^{i}d\varepsilon_{ij}^{i}\right)^{1/2},\\
\beta=-\frac{1}{3}\arcsin\frac{\sqrt{6}\,\tau_3}{\tau_2^{3/2}},\ -\frac{\pi}{6}\leqslant\beta\leqslant\frac{\pi}{6}.
\end{gathered}$$

The function $\varphi(\beta)$ has the form (2.1). The parameters k, k_1, k_2, B, n, and m are material constants. The values of B, n, and m were found from tests conducted in uniaxial creep (it is assumed that the strain-hardening law $\dot{\varepsilon} = B\sigma^n\varepsilon^m$ is valid), while k_1 and k_2 were determined in [14] (for example) by replotting empirical data in the axes $\tan\omega \sim \xi$. The method of finding k is explained below. Comparison of the theory with tests [5] conducted in complex loading for material Ti6Al4V were performed under the following conditions: creep was realized with $\sigma_i = (3\sigma_{ij}'\sigma_{ij}'/2)^{1/2}$ = const and $\theta = \arctan(\sqrt{3}\tau/\sigma)$ = const; then, at t_*, the stress vector was rotated through the angle $\Delta\theta$ while leaving σ_i = const. For this material, B = $7\cdot 10^{-20}$, n = 6, m = −3, $\varphi(\beta) = 1$. The parameter k was chosen so that the curve A(t) (A is the work done during creep, $A=\int_0^t \sigma_{ij}\,d\varepsilon_{ij}^{i}$) coincided on the section $A(t_* + 0)$ with the experimental curve. Thus, k = 1450 for the given material.

Figure 1 shows graphs A ~ t, where 1 is the experimental curve from steady loading (σ_i = 637 MPa = const, θ = 0 = const); 2 is the experimental curve from complex loading (σ_i = 637 MPa = const, $\theta(t_* - 0) = 0$, $\theta(t_* + 0) = 90°$), 3 is the calculation of complex loading based on the proposed theory (k = 1450 = const). Let us look at the example in [4].

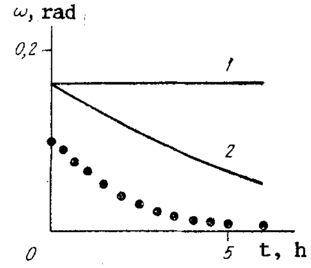

Fig. 3

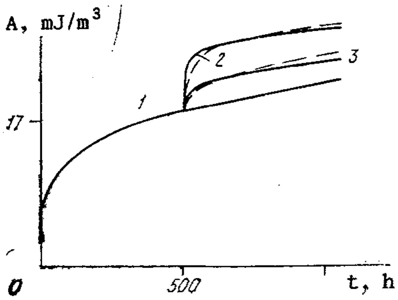

Fig. 4

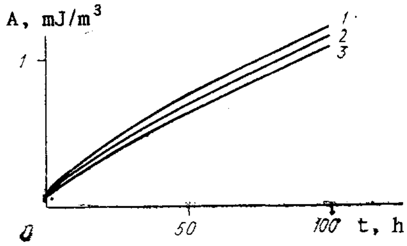

Fig. 5

Here, with a change in θ over the period of time t_* (after steady loading at σ_i = 230 MPa, θ = 60°), σ_i decreases to such a value that the graph A(t) remains unchanged for a certain period of time (compared to steady loading). In the next example, θ was changed again (after steady loading) and a new value of σ_i was chosen so that $A(t_* - 0) = A(t_* + 0)$. The surface obtained as a result in the stress space is shown by curve 2 in Fig. 2 ($B = 7 \cdot 10^{-34}$, n = 18, m = −2, $\varphi'(\beta) = 1$, k = 2000). It can be referred to as the equal-creep-rate surface [4, 5]. The circles show experimental data obtained by the above-described scheme in [4]. It is evident that the instantaneous creep surface changes form, and this agrees with the empirical data. At k = 0, the indicated surface remains circular (curve 1). It was established from the tests in [4, 5] that at σ_i = const and varying θ, the deviators $d\varepsilon_{ij}{}^i/dt$ and σ_{ij}' have a similarity phase. Over time, this phase approaches zero. In calculations performed in connection with the above-described tests conducted at k = const, a similarity phase was clearly detected but remained constant.

Figure 3 shows results of calculations of the similarity phase ω on the basis of the test method in [4] (σ_i = const, while $\Delta\theta$ was varied). Here, $\omega = \alpha - \xi$, where ξ is the angle of the deviator σ_{ij}', α is the angle of the deviator $d\varepsilon_{ij}{}^i/dt$, $k = k(\beta) = A_1 + A_2\beta$, A_1 = 2000, A_2 = −1000, and β is the angle of the deviator τ_{ij}. Figure 3 shows results of calculation of ω in accordance with the theory: line 1 corresponds to $\Delta\theta$ = 30° at k = const, line 2 corresponds to $\Delta\theta$ = 30° at $k = k(\beta)$, and the circles represent approximate empirical data from [5].

In addition, it is evident from Fig. 1 (curve 3) that the "tail" of the graph A(t) lies above empirical curve 2. To eliminate this "defect" in the theory, a suggestion has been made proposing the use of $k(\lambda) = C\exp(-\kappa\lambda) + D$ (κ = 170, D = 250, $C = 1.351 \cdot 10^6$). The results of calculations performed by this invariant are shown in Fig. 1 by the dashed line. The proposition $k(\lambda, \beta) = (C\exp(-\kappa\lambda) + D)(1 + A_3\beta)$ refines the behavior of the similarity phase.

We also examined another form of relationship between the tensors ρ_{ij} and $\varepsilon_{ij}{}^i$:

$$d\rho_{ij}/d\lambda + a\rho_{ij} = b(\beta)\, d\varepsilon_{ij}^{\mathrm{i}}/d\lambda.$$

Then the working formulas (3.1) takes the form

$$d\varepsilon_{ij}^{i}/dt = B\left[\sqrt{\tau_2}\,\varphi(\beta) + \sqrt{\rho_2}\right]^n \lambda^m L_{ij},$$
$$d\rho_{ij}/dt = b(\beta)\, d\varepsilon_{ij}^{i}/dt - a\rho_{ij}\, d\lambda/dt,\; d\lambda/dt = B\left[\sqrt{\tau_2}\,\varphi(\beta) + \sqrt{\rho_2}\right]^n \lambda^m. \quad (3.2)$$

Here $\rho_2 = \rho_{ij}\rho_{ij}$; a = const, $b(\beta) = B_1 + B_2\beta$.

Figure 4 shows the results of calculations performed with Eqs. (3.2) for $\sigma_i = 637$ MPa = const [curve 1) $\theta = 0$, $\Delta\theta = 0$, 2) $\Delta\theta = 90°$, 3) $\Delta\theta = 30°$, dashed lines – results calculated from the theory with a = 3, b(β) = const = 1400]. The similarity phase of the deviators $d\varepsilon_{ij}^{i}$ and σ_{ij}' is fixed and decreases slowly with time.

Figure 5 shows data from the calculation of the work done in the creep process during the steady loading of material AK4-1T. In Eq. (2.1), $k_1 = 0.0505$ and $k_2 = 0.1364$ for this alloy. Calculations performed by the simplest variant of the theory produce a "split" of the graphs A(t) with σ_i = const for different θ (curves 1-3 for θ = 90, 60°, and 0). They also yield a constant similarity phase for the deviators of creep strain rate and stress during steady loading. This finding is consistent with the test results.

Thus, the theory formulated here makes it possible to describe a number of interesting effects relating to the creep of materials which are not sensitive to the type of stress state and react to the mode of loading – whether the loading is steady or complex.

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